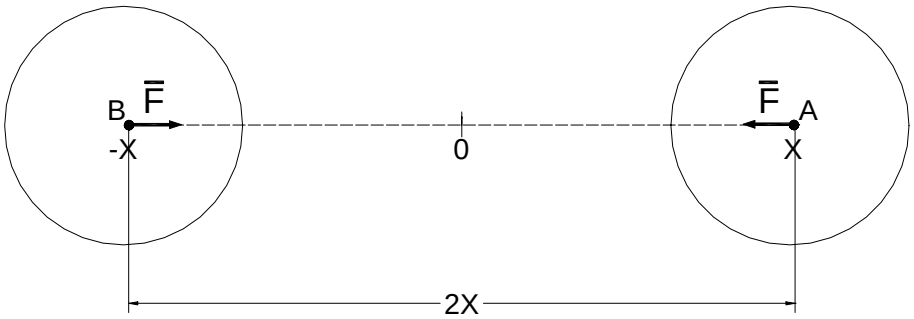
Oefening 19:



Newton: $\mathbf{F = K\frac{M_1M_2}{r^2} = K\frac{m^2}{(2X)^2}}$

Bewegingsvergelijking voor rechtersfeer: $\mathbf{m\ddot{X} = -F}$

$$\mathbf{\Rightarrow \ddot{X} = -\frac{Km}{4}\frac{1}{X^2}} \qquad \mathbf{|.\dot{X}} \qquad \textbf{en integreren}$$

$$\mathbf{\Rightarrow \int \dot{X}\ddot{X}dt = -\frac{Km}{4}\int \frac{\dot{X}}{X^2}dt}$$

$$\mathbf{\Rightarrow \frac{1}{2}\int d(\dot{X}^2) = -\frac{Km}{4}\int \frac{1}{X^2}dX}$$

$$\mathbf{\Rightarrow \frac{1}{2}\dot{X}^2 = \frac{Km}{4}\frac{1}{X} + C}$$

$\mathbf{t = 0:}$ $\quad \mathbf{X_0 = 2a} \qquad \mathbf{\dot{X}_0 = 0}$

$$\mathbf{\Rightarrow \quad C = -\frac{Km}{8a}}$$

$$\mathbf{\Rightarrow \quad \dot{X}^2 = \frac{Km}{4a}\left(\frac{2a}{X} - 1\right)}$$

$$\mathbf{\Rightarrow \quad \frac{dX}{dt} = \pm\sqrt{\frac{Km}{4a}}\sqrt{\frac{2a - X}{X}}}$$

$$\mathbf{\Rightarrow \quad t - t_0 = \pm\sqrt{\frac{4a}{Km}}\int_{2a}^{X}\sqrt{\frac{X}{2a - X}}dX}$$

stel $y^2 = \dfrac{X}{2a - X} \quad \Rightarrow \quad X = 2a\dfrac{y^2}{1+y^2}$

$$\Rightarrow \quad dX = 4a\frac{y}{(1+y^2)^2}dy$$

$X:$ $\underbrace{a}_{\text{botsing}}$, $\underbrace{2a}_{\text{begin}}$ $\Rightarrow$ $y:$ $1,$ ∞

$$\Rightarrow \quad t^* = \pm\sqrt{\frac{4a}{Km}}\int_{\infty}^{1}\frac{4ay^2}{(1+y^2)^2}dy = \mp 2a\sqrt{\frac{4a}{Km}}\int_{1}^{\infty}\frac{2y^2}{(1+y^2)^2}dy$$

en $\displaystyle\int\frac{2y^2}{(1+y^2)^2}dy = -\frac{y}{1+y^2} + \text{bgtg}\, y$

$$\Rightarrow \quad t^* = \mp 4a\sqrt{\frac{a}{Km}}\left[-\frac{y}{1+y^2} + \text{bgtg}\, y\right]_1^{\infty}$$

$$\Rightarrow \quad t^* = \mp 4a\sqrt{\frac{a}{Km}}\left(\frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad t^* = +\,a\sqrt{\frac{a}{Km}}\,(\pi + 2)$$

en voor een massa aan het aardoppervlak geldt:

$$m\ddot{Y} = K\frac{mM_A}{R_A^2}$$

$$\ddot{Y} = g \qquad \Rightarrow \quad KM_A = gR_A^2$$

en:

$$M_A = \frac{4}{3}\pi\rho_A R_A^3 \qquad \Rightarrow \quad K = \frac{3g}{4\pi\rho_A R_A}$$

en :

$$m = \frac{4}{3}\pi\rho a^3 \qquad \Rightarrow \quad t^* = \sqrt{\frac{R_A\rho_A}{g\rho}}(\pi + 2)$$

numeriek voorbeeld:

$$t^* = (\pi + 2)\sqrt{\frac{6400.10^3}{9.81}.\frac{4}{5}} = 3716.38\ \text{s}$$