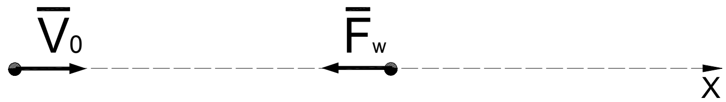
Oefening 12:



Bewegingsvergelijking: $\quad m\ddot{X} = -F_W = -\alpha \dot{X}^2$

Stel $\quad \dot{X} = V$

a) $\quad \dfrac{dV}{-\dfrac{\alpha}{m} V^2} = dt \qquad$ integreren

$$\frac{m}{\alpha}\left(\frac{1}{V} - \frac{1}{V_0}\right) = t - \underbrace{t_0}_{=0}$$

$$\frac{1}{V} = \frac{\alpha}{m} t + \frac{1}{V_0} = \frac{\alpha V_0 t + m}{m V_0} \qquad \Rightarrow \quad V = \frac{m V_0}{\alpha V_0 t + m}$$

b) $\quad dX = \dfrac{m V_0}{\alpha V_0 t + m} dt \qquad$ integreren

$$\Rightarrow \quad X - \underbrace{X_0}_{=0} = \frac{m V_0}{\alpha V_0} \ln\left|\frac{\alpha V_0 t + m}{\alpha V_0 \underbrace{t_0}_{=0} + m}\right| \quad \Rightarrow \quad X = \frac{m}{\alpha} \ln\left|\frac{\alpha V_0 t + m}{m}\right|$$

$$\Rightarrow \qquad X = \frac{m}{\alpha} \ln\left|1 + \frac{\alpha}{m} V_0 t\right|$$

c) $\quad \ddot{X} = \dfrac{dV}{dt} = -\dfrac{m V_0}{(\alpha V_0 t + m)^2} \alpha V_0 = -\dfrac{m \alpha V_0^2}{(\alpha V_0 t + m)^2}$

d) $\quad X = \dfrac{m}{\alpha} \ln\left|1 + \dfrac{\alpha}{m} V_0 t\right|$

$$\Rightarrow \quad 1 + \frac{\alpha}{m} V_0 t = e^{\frac{\alpha}{m} X}$$

$$\Rightarrow \quad t = \frac{m}{\alpha V_0}\left(e^{\frac{\alpha}{m} X} - 1\right) \quad \text{substitueren in } V = \frac{m V_0}{\alpha V_0 t + m}$$

$$\Rightarrow \quad V = \frac{m V_0}{m\left(e^{\frac{\alpha}{m} X} - 1\right) + m} = \frac{V_0}{e^{\frac{\alpha}{m} X}} = V_0 \, e^{-\frac{\alpha}{m} X}$$

e) $\quad \ddot{X} = -\dfrac{m \alpha V_0^2}{m^2\left(e^{\frac{\alpha}{m} X} - 1 + 1\right)^2} = -\dfrac{\alpha}{m} V_0^2 e^{\frac{-2\alpha}{m} X}$